Teacher notes
Topic C

Forced oscillations.

A block of mass *m* is attached to a spring obeying Hooke's law with spring constant *k*.

(a) Show that the extension of the spring at equilibrium is given by $e = \frac{mg}{k}$.

(b) The mass is pulled a small distance x_0 below the equilibrium position and is then released. Explain why the mass will perform simple harmonic oscillations with angular frequency given by $\omega = \sqrt{\frac{k}{m}}$.

(c) Write down the equation giving the displacement of the block from equilibrium. Take the down direction to be positive.

(d) A cardboard is attached to the block so that the oscillations are now lightly damped. The displacement is given by $x = x_0 \sin(2\omega t + \frac{\pi}{2})$. State and explain **two** reasons why a periodic force acts on the system.

(e) The angular frequency of the periodic force is increased. State and explain the effect of this change on the amplitude of oscillations.

Answers

(a) At equilibrium $ke = mg \Rightarrow e = \frac{mg}{k}$.

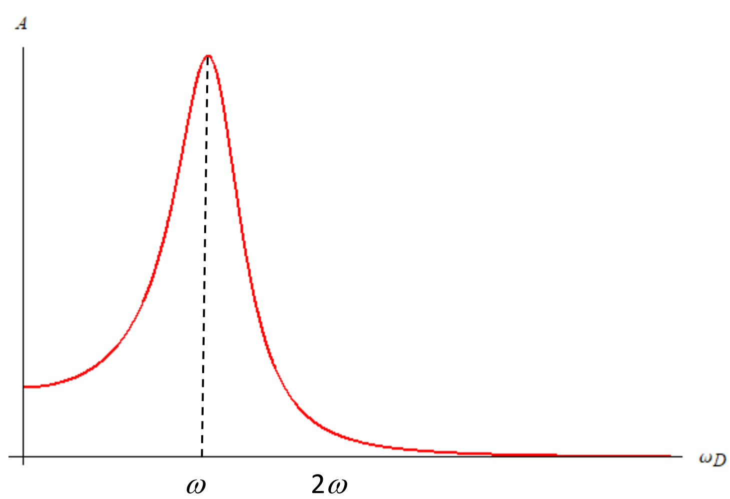
(b) When extended *downwards* by a distance *x* below the equilibrium position the **net** force on the block is $k(e+x) - mg = ke + kx - mg = kx$ *upwards*. The net force is thus opposite to the displacement and proportional to it, so SHM takes place. $ma = -kx \Rightarrow a = -\frac{k}{m}x \Rightarrow \omega^2 = \frac{k}{m}$.

(c) $x = x_0 \sin(\omega t + \frac{\pi}{2})$.

(d)

- If there was no external periodic force acting the amplitude would be decreasing exponentially but here it is constant.
- The angular frequency of oscillations is 2ω and is different from the natural frequency ω indicating that there is an external force of frequency 2ω.

(e) Recall that the variation of the amplitude with external force angular frequency is given by:



Since we now have a force with frequency 2ω, increasing the frequency further means the amplitude will decrease.